The Random Phase Approximation (RPA) and GW approximation for electrochemistry

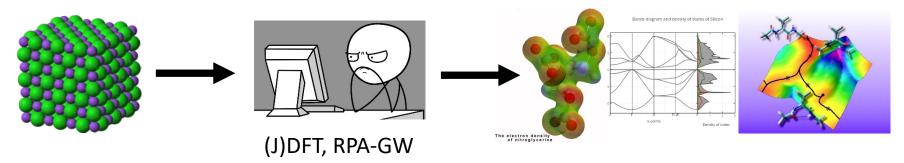
The BEAST collaboration 3rd Annual BEAST Workshop August 23, 2024



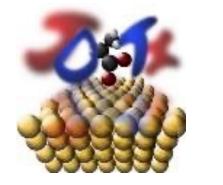
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Density functional theory (Kohn-Sham)



- Predict ground state properties without empirical input
- Properties: structure, charge density, reaction energetics
- Thousands of practitioners worldwide (16k citations on Hohenberg, Kohn)
 - JDFTx, Quantum ESPRESSO, VASP, ABINIT, EXCITING, Fleur, Octopus, ELK





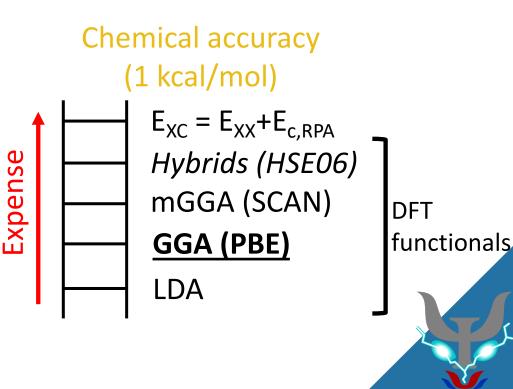


Density functional theory (Kohn-Sham)

• Describing exchange-correlation term is key

$$\left(-\frac{1}{2}\nabla^2 + V_{ion} + V_{hartree}[n] + V_{xc}[n]\right)\psi_{nk} = \varepsilon\psi_{nk}$$
$$E_{DFT} = T_{KS} + E_{ion-el} + E_H + E_{XC}$$

- Calculate E_{XC} with varying degrees of accuracy/expense
 - 0 K, vacuum conditions
- What functional gives accurate energies and electronic structure... generally?





Adiabatic Connection Fluctuation Dissipation Theorem (ACFDT)

• Non-interacting Hamiltonian constructed such that its ground state Slater determinant $|\phi_0\rangle$ gives same density as true ground state wavefunction $|\psi_0\rangle$

• Generalization of Kohn-Sham DFT: scale the coulomb interaction

$$v_c \to \lambda v_c \qquad \qquad 0 \le \lambda \le 1$$

and obtain ground state wavefunction wavefunction $|\psi_0^{\lambda}\rangle$ gives same density as true ground state wavefunction $|\psi_0\rangle$

$$|\psi_0^{\lambda=0}\rangle = |\phi_0\rangle$$
 and $|\psi_0^{\lambda=1}\rangle = |\psi_0\rangle$



Key quantity: interacting polarizability χ^{λ}

• Solve Dyson equation

$$\chi^{\lambda}(\omega) = \chi_{\rm KS}(\omega) + \chi_{\rm KS}(\omega) \left[\lambda v_{\rm c} + f_{\rm xc}^{\lambda}(\omega)\right] \chi^{\lambda}(\omega)$$

- The RPA: $f_{xc}^{\lambda} = 0$
- Obtain polarizability
- Can calculate RPA correlation energy analytically

$$E_c^{RPA} = \int_0^\infty \frac{d\omega}{2\pi} \operatorname{Tr} \Big[\ln\{1 - \chi^0(i\omega)v\} + \chi^0(i\omega)v \Big]$$

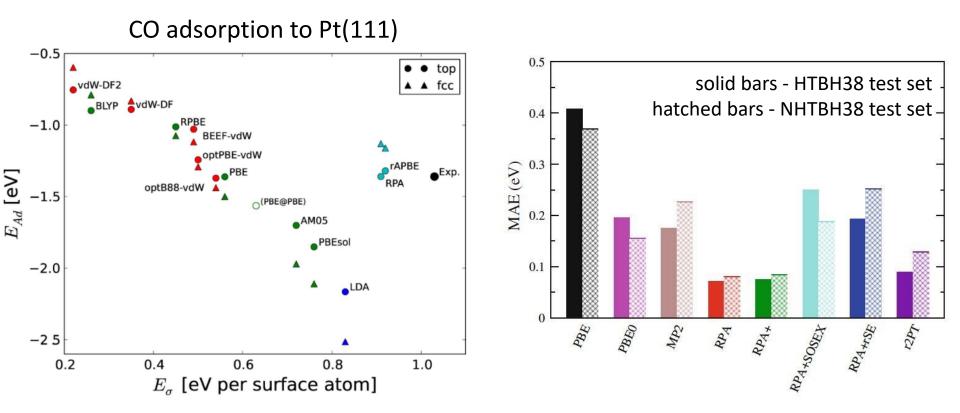


RPA

• Features

- RPA gives correlation energy, compute EXX using input orbitals
- Exactly cancels self-interaction from Hartree term (some self-interaction in correlation)
- O(N4) complexity: between DFT and high-fidelity MP2, CCSD, CCSD(T)
- Parameter-free dispersion forces: key for surface chemistry, binding of 2D materials
- Shortcomings
 - So-so short-range correlation: inaccurate cohesive energies, atomization energies
 - Slow convergence with kinetic energy cutoff (wavefunction cusp condition)

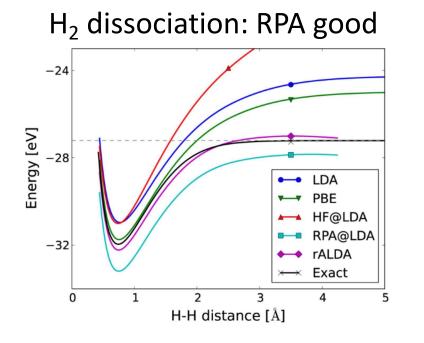
RPA can predict accurate properties relevant to catalysis



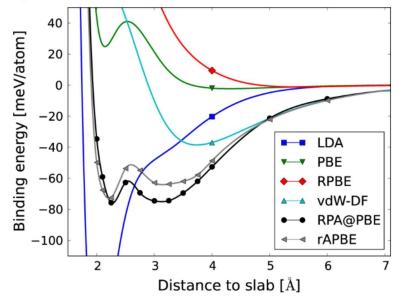
HTBH38: forward, inverse barrier heights of 19 hydrogen-transfer reactions, NHTBH38: 19 reactions involving heavy atom transfers, nucleophilic substitutions, association, unimolecular processes.

Olsen, et al., npj Computational Materials, 5 (2019) Ren, et al., J. Mat. Sci., 47 (2012)

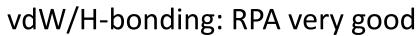


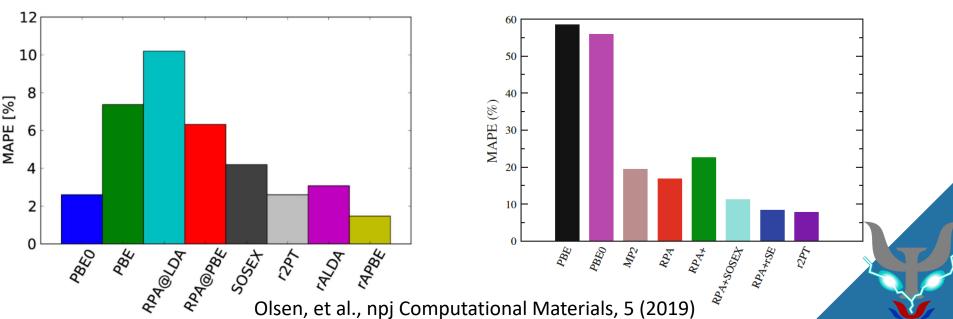


Graphene*+Ni(111): RPA good



Atomization energies: RPA ok





What is the GW approximation?

• Dyson equation: replace exchange-correlation potential with many-body self-energy

$$\left[\frac{\nabla^2}{2} + V_{ion} + V_H + \Sigma \left(E_{nk}^{QP}\right)\right] \psi_{nk}^{QP} = E_{nk}^{QP} \psi_{nk}^{QP}$$

 $\Sigma = iGW$

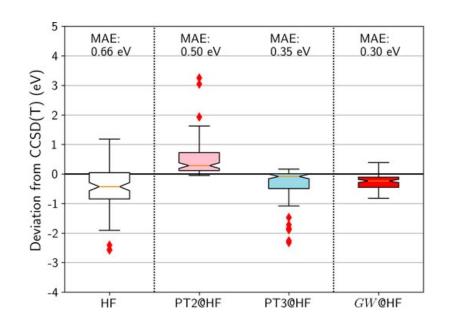
$$W_{GG'}(q;\omega) = \epsilon_{GG'}^{-1}(q;\omega)\nu(q+G')$$

$$\epsilon_{GG'}(q;\omega) = \delta_{GG'} - \nu(q+G)\chi^0_{GG'}(q;\omega)$$

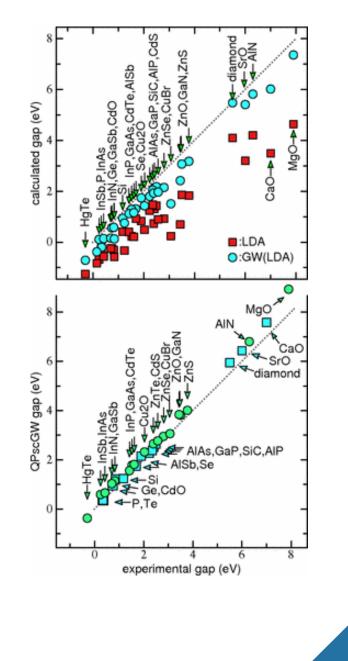
Polarizability calculated within random phase approximation



GW can predict accurate IPs and band gaps



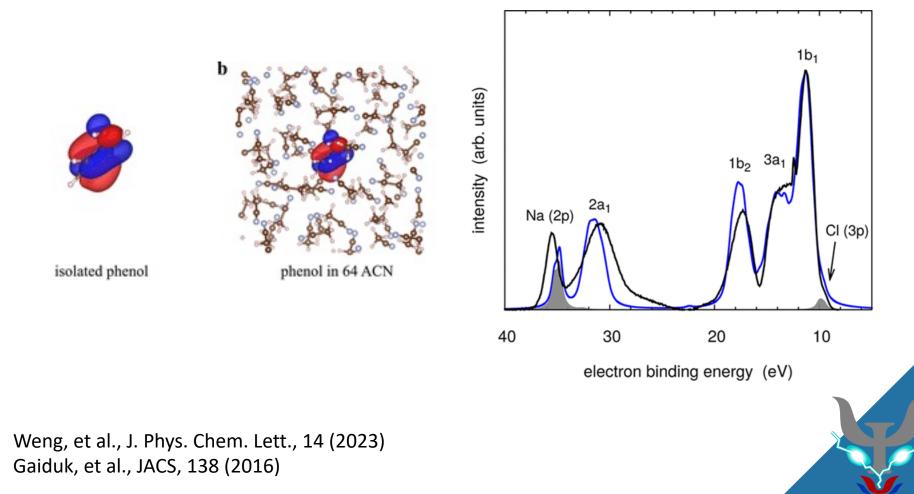
Bruneval, et al., Front. Chem. 9 (2021) van Schilfgaarde, et al., PRL, 96, 226402 (2006)



Solvated beyond-DFT calculations are non-trivial

- Solvated GW calculations have previously used AIMD sampling of explicit waters or simplified implicit solvation models
 - 1 M NaCl = 1 Na + 1 Cl + 52 H₂O

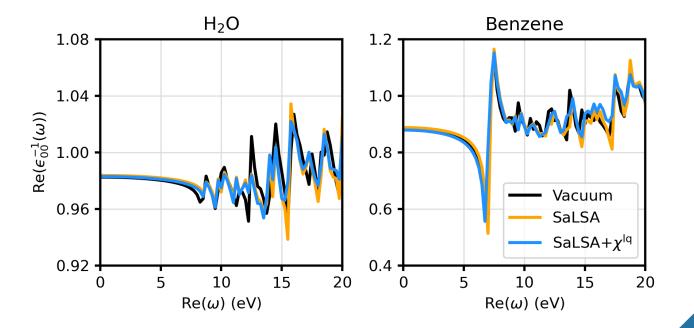
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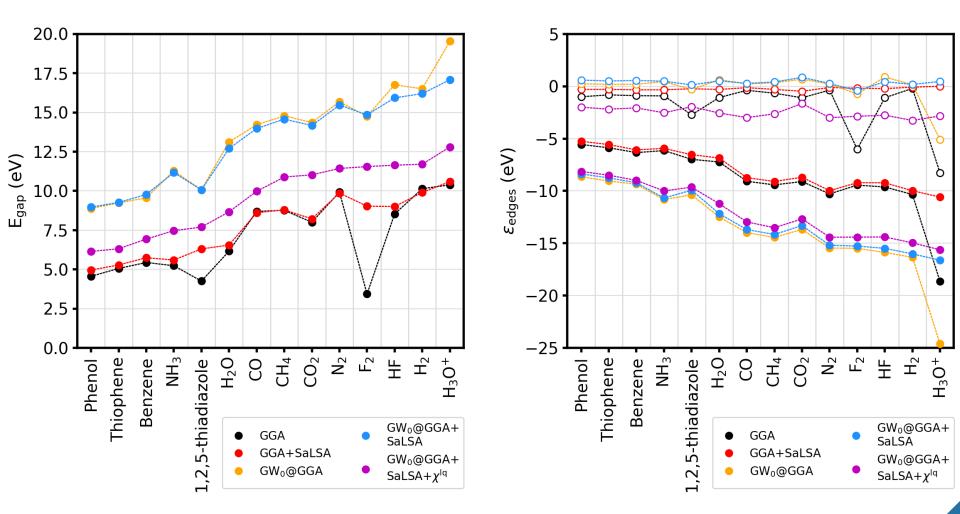
Merging GW with advanced implicit solvation models

- We published a formalism merging GW with SaLSA model
 - J Clary, M Del Ben, R Sundararaman, D Vigil-Fowler, "Impact of solvation on the GW quasiparticle spectra of molecules", *J. Appl. Phys.* 134, 085001 (2023)
- Use wavefunctions from solvated DFT calculation
- Combine fluid polarizability with electronic polarizability from SaLSA

$$\chi^{solv} = \chi^0 + \chi^{lq}$$



Get lowering of HOMO-LUMO gap in molecules





Get IP agreement with experiment where available

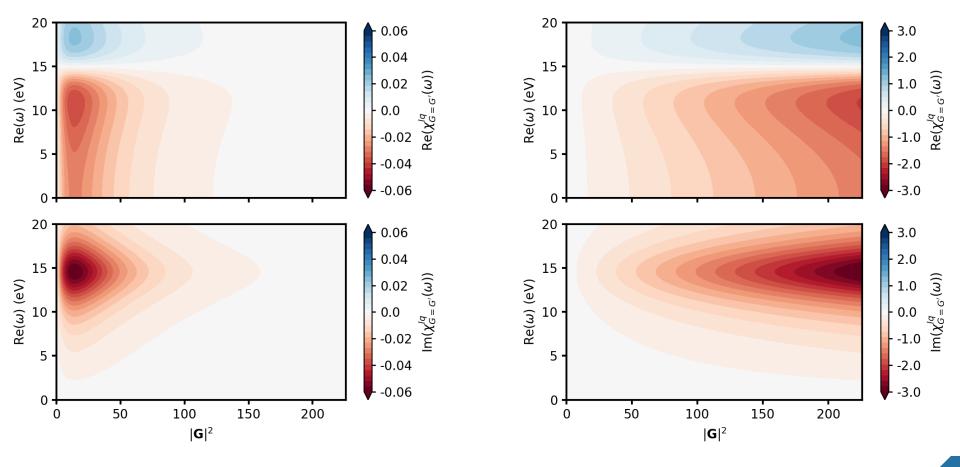
Molecule	IP	$egin{array}{l} { m GW_0@GGA+} & { m SaLSA+}\chi^{ m lq} & \ { m IP \ (eV)} \end{array}$	Experimental IP (eV)
H_2O	1^{st}	11.224	$11.1 - 11.5^{58-61}$
H_2O	2^{nd}	13.468	$13.5 - 13.8^{59-61}$
H_2O	3^{rd}	17.339	$17.3 - 17.4^{59,61}$
H_3O^+	2^{nd}	20.619	$20 - 21^{62}$
Phenol	$1^{\rm st}$	8.141	$7.8 - 8.3^{63,64}$
Phenol	2^{nd}	8.924	8.6^{63}



Have to use SaLSA currently to avoid divergence

SaLSA

GLSSA13





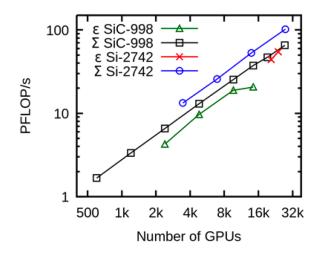


Foreseen exascale HPC systems will be GPU accellerated architectures

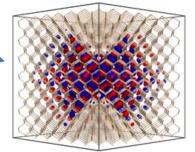
Optimized version of BerkeleyGW on GPU accelerated systems:

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- Scale up to the full Summit machine at OLCF: >27k GPUs
- Reach nearly 53% of the peak performance at 106 PFLOP/s
- Time to solution of ~10 mins for 11k electrons system



M. Del Ben, C. Yang, Z. Li, F. H. da Jornada, S. G. Louie and J. Deslippe, "Accelerating Large-Scale Excited-State GW Calculations on Leadership Class HPC Systems" in Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis, ser. SC '20 No.4 pp.1 (2020), ACM Gordon-Bell Finalist



Divacancy defect in semiconductor (such as Si and SiC) are proxy for solid state Qubits. For silicon shown is the 2742-atoms Si supercell, 10,968 electrons.